1 Important Dates

Proj 6 due Friday 22 Nov
Proj 7 due Friday 6 Dec
is final project: pledged
Lab for proj 6 on Wed 20 Nov

2 Makefiles in cppvectorpkg

In src/cppvectorpkg/
Makefile is pretty basic: assumes all header files
in main/
Makefile uses relative path names
makes your structure rigid
things have to be in the same location every time
will work for the project, we set the structure of our directory
graders will just move a copy of the directory
don't include CLAPACK in directory
there are 2 libraries in CLAPACK, but need to make it look there: need
to find the correct library in make by making sure they are
called the right thing: liblapack_LINUX.a and libblas_LINUX.a

3 Least Squares

matrix A (mxn), m not necessarily equal to n
Ax=b might not be solvable for x, or not uniquely solvable
Gaussian elimination is not going to solve it
Since we can't assume that there is a solution, we can try:
minimize (Ax-b).
Choose x so that $J(x) = (Ax-b)^T(Ax-b) = \sum_{i=1}^{m}(Ax-b)_i^2$ is a minimum.
This isn't a linear system, which we can solve, but it is equivalent to one.
J is a quadratic polynomial in x.
$J(x) = (Ax)^T Ax - b^T Ax - (Ax)^T b + b^T b$
$= x^T A^T Ax - 2x^T A^T b + b^T b$ since $b^T Ax$ and $(Ax)^T b$ are scalars
$= \sum_l(\sum_k(\sum_j(x_l A_{kl} A_{kj} x_j))) - 2 * \sum_l(\sum_k(x_l A_{kl} b_k + b'b))$
If x minimizes J, then
grad J(x) = 0

$(gradJ(x))_i = \frac{\delta}{\delta x_i}(J(x))$
$= \sum_k(\sum_j(A_{ki}A_{kj}x_j)) + \sum_l(\sum_k(x_l A_{kl} A_{ki})) - 2 * \sum_k(A_{ki}b_k)$
l and j are both 1:n indexers, if you rename l to j:
$(gradJ(x))_i = \frac{\delta}{deltax_i}(J(x))$
$= 2 * \sum_k(\sum_j(A_{ki}A_{kj}x_j)) + -2 * \sum_k(A_{ki}b_k)$
$= 2 * (A^T Ax - A^T b)_i = 0$ when $A^T Ax - A^T b = 0$
So, miminize $J(x) ==>$ find a solution x of $A^T Ax = A^T b$
This is a square linear system. HOORAY!
This equation is known as the normal equation(s)
Also, all solutions of the normal equation(s) are minimizers of J.
$A^T A$ is symmetric and positive semidefinite: $x^T A^T Ax >= 0$ for any x since it just just sums of squares
Since it's positive, and since it's basically the 2nd derivitave of J, it causes all extrema of J to be a minimum.
If $A^T A$ is positive definite, $x^T A^T Ax > 0$ unless x=0
==> $A^T A$ is invertible
==> normal equation(s) have a unique solution
==> unique stationary point of J which is the (global) minimizer.
The structure of $A^T A$ lets you solve this a lot more efficiently
CLAPACK has a few things that allow you to take advantage of this.