

CAAM 420 Fall 2011 Homework 2

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You must complete the following task by 5pm on Thursday 09/08/11.

Your solutions to the homework must be committed to your Subversion repository in a sub-directory HW02. You will commit two files:

- i. a text write up formatted as a pdf file in HW02/hw02.pdf
- ii. your source code file in HW02/hw02.c

Q1 *Purpose: practicing programming, testing and debugging simple mathematical functions in C*

The sine function can be evaluated using its Taylor series:

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} +$$

which in summation notation is

$$\sin(x) = \sum_{n=0}^{n=\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}.$$

Unfortunately we cannot directly evaluate an infinite sum in finite time. Thus we choose to evaluate the partial sum to approximate the infinite sum:

$$s_N(x) = \sum_{n=0}^{n=N} \frac{(-1)^n x^{2n+1}}{(2n+1)!}.$$

Homework tasks:

- i. [10 points] Use the mean-value form of the remainder of the $(N + 1)$ term Taylor expansion for the sine function to write down an upper bound for

$$\epsilon_N(x) := |\sin(x) - s_N(x)|,$$

in terms of x and N that can be evaluated without explicitly evaluating $\sin(x)$.

- ii. [0 points] Create a C-file called hw02.c with emacs.
- iii. [30 points] In hw02.c write a function mysin that takes two arguments and has prototype:

```
double mysin(double x, double tol)
```

Implement mysin so that it:

- chooses N such that $\epsilon_N(x) < \text{tol}$
- computes the partial sum $s_N(x)$
- returns the value of $s_N(x)$.

Warning: `mysin` must not use the standard math library function `sin`.

iv. [30 points] In `hw02.c` create a `main` function that evaluates `sin` and `mysin` for

- a. $x = 0.2$,
- b. $x = -4.5$,
- c. $x = 120.3$,

using $tol = 1e - 5$ for the `mysin` tolerance. In each case compute the difference between the outputs of `mysin` and `sin`.

v. [10 points] Document your results as a pdf formatted paper stored in `HW02/hw02.pdf`. Explain any deviation of the output from your `mysin` function when compared to the `sin` function.

vi. [10 points] Describe briefly the debugging process you used to make sure your `mysin` function was behaving as specified.

vii [10 points] Add and commit your `hw02.c` and `hw02.pdf` files to a subdirectory `HW02` directory in your CAAM420 Subversion repository.

viii [Extra credit: 20 points] Improve the computation of `mysin(x)` for large x . Explain your approach and report any change in the results for part [iv.]

