CAAM 420 Fall 2011 Homework 7

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November 19, 2011

You must complete the following task by 5pm on Friday 12/02/11.

Your solutions to the homework must be committed to your Subversion repository in a sub-directory `HW07`.

You may work in a group of one or two students. If you work alone then you may choose Option 1 or Option 2. If you work in a group of two your group must complete both Option 1 and Option 2.

All source code, header files, LaTeXfiles, Makefiles specific to Option 1 should be committed to your own repository under `HW07/Option1`. Similarly all source etc specific to Option 2 should be committed to `HW07/Option2`. If you use a common code base for both Options then you should commit those files under `HW07/Common`, with the Option specific codes in the `HW07/Option1` or `HW07/Option2` directories.

Use LaTeXand Kile to write and typeset your report (saved as `HW07/report.pdf` in your Subversion repository). Where diagrams are requested you should use MATLAB or InkScape link to create vector graphics to include in your report.

If you are working in a team of two you may discuss the effort with the other member of your team. Otherwise you may only consult the instructor or graders for verbal assistance. You are encouraged to use textbooks and internet resources. You must cite all resources used via footnotes or a bibliography.

150 points will be awarded for a successful completion.
Extra credit will be awarded as appropriate.

Q1 *Build a sparse matrix representation of a resistor network. Use iterative methods to solve for the circuit node potentials or circuit loop currents.*

Problem description: The file circuit1.msh contains a gmsh generated mesh of quadrilateral elements. You should view this as a resistor network. Each edge of each element corresponds to a wire in the network. When two elements share the same edge, i.e. both contain the same pair of nodes in the graph then we assume that the wire includes a resistor of 1 Ohm resistance. When an element does not share the edge with another element we assume a resistor on that wire of 0.5 Ohms resistance. See Figure 1 for an illustration.

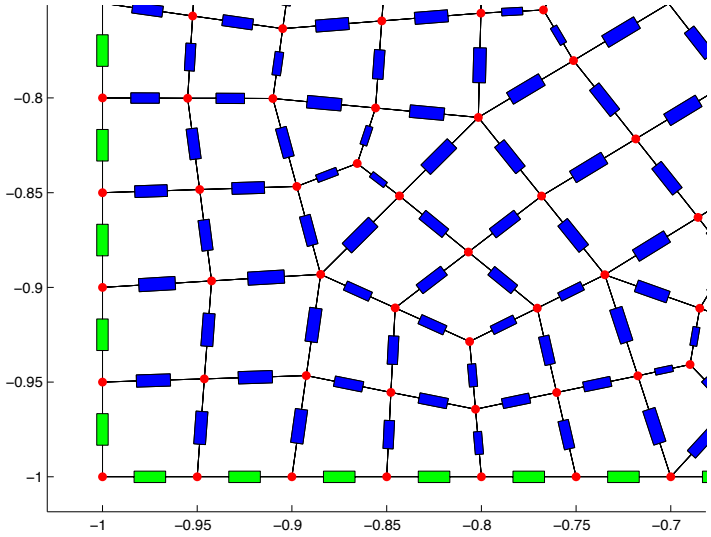

Figure 1: Corner of resistor network. Red circles: nodes. Blue rectangles: 1 Ohm resistors. Green rectangles: 0.5 Ohm resistors. Black lines: lossless wires.

Task:

Your task is to create a C++ code that reads in a given mesh file (gmsh .msh format) then builds a circuit matrix and load vector (representing the voltage source) that can be used to determine the current in each wire of the resistor network, assuming that you attach a voltage source to one of the wires.

You are expected to use a C++ sparse matrix class to store the circuit matrix. The load vector and solution vector should be stored with a dense storage convention.

The details are up to you. However, it is recommended that you consider using the skeleton code available for download at HW07skeleton.tar.gz.

Options: There are two options you can choose from to solve for the currents in each wire:

- Option 1: *method of node potentials*
  Choose to represent the unknowns in the circuit as voltage potentials at the mesh nodes. You should choose one node, likely the highest numbered node, to have zero potential. After computing the node potentials you should Ohm's law to find the current in each wire.

- Option 2: *method of loop currents*
  Choose to represent the unknowns in the circuit as loop currents. In this representation you will associate a current with each quadrilateral element. After computing the loop currents you will reconstruct the current in each wire by combing the loop currents of the one or two elements that have the wire as an edge.

Solver: In each Option you are allowed to use an iterative solver to solve the linear system to find the unknowns. Choose at least two from:

A. Jacobi method wiki.

B. Gauss-Seidel method wiki.

C. Successive over relaxation wiki. You will have to determine a reasonable relaxation parameter.

D. Conjugate gradient method wiki.

Convergence: It is possible that one or more of these methods will not converge to the solution. CAAM students should verify the sufficiency conditions for convergence are satisfied by the circuit matrices.

Compare the number of iterations and number of operations that your two chosen methods require.

[Ninja extra credit:] The convergence for A-C is likely to be slow, taking 1000s of iterations to drop the equation residual a few decimal places. Adventurous students can investigate if a multigrid solver yields better performance.

Test case: Design a simple test case that you can verify manually and make sure that your solver yields the correct wire currents. Use Inkscape to create a vector-art diagram of your simple circuit and include it in your report.

Scale up: Use gmsh to create a sequence of mesh files with 100, 1000, 10000, 100000 elements. Is your circuit solver able to handle these sized circuits ? Give some timing results.