Topology and Neuroscience: Data analysis

Yuri Dabaghian
Baylor College of Medicine & Rice University
dabaghian at rice.edu
Hippocampus and space coding

- Forming new memories

- Spatial behavior, spatial memory
  - navigation, navigation planning
  - imagining spatial scenes
Electrophysiology of spatial coding
Electrophysiology of spatial coding
Electrophysiology of spatial coding

Firing activity is correlated with spatial location
Hippocampal spatial map

~ 4,000 cells are active simultaneously on each side

~400,000 cells total on each side

The ensemble of place cells forms a map of the environment
A space with 2 holes
Environment with 2 holes + trajectory
Examples of simulated place fields

Place field center, $\vec{r}_0$

Maximal firing rate, $F$

Width of the distribution, $s$

Firing probability distribution

$$f(\vec{r}) = Fe^{-\frac{(\vec{r}-\vec{r}_0)^2}{s^2}}$$
A place field map of a 2 hole environment

An ensemble of $N$ place cells forms a map of the environment

Information transmitted by the place cells to the downstream neurons
A schematic structure of the place field map

Information transmitted by the place cells to the downstream neurons
Topology from neural networks

“Place cells”

“Place fields”

Eichenbaum, 2007
Topological analysis - Čech complex

Space $X$

Simplicial complex $K$

Space cover

Homologies $H_q(K) \sim H_q(X)$
Betti index – base cycles in every dimension

“Topological barcode”

- **Circle**: $(1, 1, 0, 0,...)$
- **Sphere**: $(1, 0, 1, 0,...)$
- **Torus**: $(1, 2, 1, 0,...)$
Čech complex - review
Čech complex - review
Čech complex - review
Čech complex - review
Čech complex - review
Čech complex - review
Čech complex - review

This is what we need
Čech complex – between space and time

Place field map

Simplicial complex
Temporal Čech complex
Temporal Čech complex
Temporal Čech complex
Temporal Čech complex
Temporal Čech complex
Temporal Čech complex
Čech complex – between space and time

Dabaghian et al., (2007)
Temporal Čech complex

Dabaghian et al., (2007)
Topological information unfolds in time

Accumulation of spikes

Accumulation of topological information

Dabaghian et al, (2008)
Temporal coactivity $\rightarrow$ place field connectedness
Can this actually work?
Can the map sustain biological variability?

Can this actually work?
Topics for topological analysis

- Is topology always extractible?
- Are the results robust?
- Does it take a reasonable time?

Answers depend on the quality of a map
Quality of the spatial map

Map characteristics:

• Firing rates of place cells
• Place field sizes
• Etc.

\[ \langle F \rangle, \langle s \rangle, N \]
Quality of the spatial map

Map characteristics:

- Firing rates of place cells
- Place field sizes
- Etc.

\[ \langle F \rangle, \langle s \rangle, N \]
Topological persistence
Topological persistence
Topological persistence

- **Points**
- **ε-Balls**
- **Čech Complex**
Topological persistence

Points

e-Balls

Čech Complex
Topological barcode of a sphere

Topological barcode \((1, 0, 1, 0, 0 \ldots)\)
Computational test of a map’s

A map model (rate and size distributions)
Computational test of a map’s

A map model (rate and size distributions)

Persistent homology, loop dynamics

Learning time → $t_{\text{min}}$
Learning time via map parameters

\[ T_{min} = T_{min} \left( \langle F \rangle, \langle s \rangle, N \right) \]

Learning time depends on map parameters

a. firing rate, \( \langle F \rangle \)
b. place field size, \( \langle s \rangle \)
c. number of cells, \( N \)
Minimal learning times $T_{\text{min}} = T_{\text{min}} (\langle F \rangle, \langle s \rangle, N)$
Minimal learning times $T_{\text{min}}$, different arenas
Place cell ensemble activity parameters

- **Firing rates**
  - Mean: 10.4503
  - Variance: 19.5334
  - Distribution: Normal

- **Place field sizes**
  - Mean: 10.4
  - Variance: 12.5
  - Distribution: Normal
Minimal learning times, $T_{\text{min}}$
1. Which parameters affect the map?

2. How does $t_{min}$ change with the biological parameters?

3. How does $t_{min}$ change with the geometry of the environment?

4. What is the “stability range” of the firing activity?

5. etc ...
Visual cortex
Visual cortex – orientation selectivity

Light bar stimulus projected on screen

Record

Stimulus orientation: [Red, Blue, Green, Yellow] presented over time (s)

Tuning curve: Cell's response based on orientation of bar

Orientation of bar: [Red, Blue, Green, Yellow]
http://homepages.inf.ed.ac.uk/jbednar/index.html
Visual cortex – orientation selectivity

\[ \theta = f(x, y) \]
Director field with topological singularities

Topological charges of the director field singularities

N. Swindale, 1996

Courtesy of V. Kalatsky, UH
Vector field with topological singularities

N. Swindale, 1996
Vector field with topological singularities
Dynamics of the vector field

Why so much geometry?
How is it used???
Topological structure of the receptive fields

Why so much topology?
How is it used???
Neuroscience and geometry
Neuroscience and geometry

The ideal world of geometry

Time code machine
...Quapropter bono christiano, sive mathematici, sive quilibet impie divinantium, maxime dicentes vera, cavendi sunt, ne consortio daemoniorum animam deceptam, pacto quodam societatis irretiant.

St. Augustine, Confessions II, xvii, 37

...Hence, a devout Christian must avoid mathematicians and all impious soothsayers, especially when they tell the truth, for fear of leading his soul into error by consorting with demons and entangling himself with the bonds of such association.
Acknowledgements

- Steve Cox, Rice University
- Harel Shouval, University of Texas
- Facundo Memoli, Stanford University
- Gunnar Carlsson, Stanford University
- Loren Frank, UCSF
- The Audience