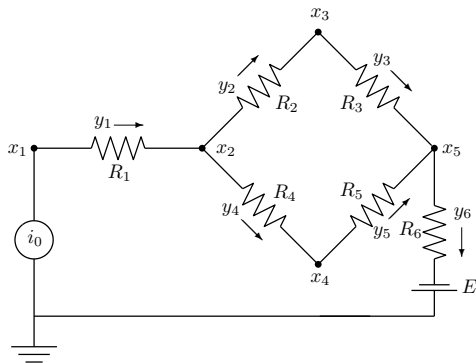


# Electrical Circuit



Set up the equations

$$e = -Ax - b,$$

Kirchhoff's voltage law (KVL)

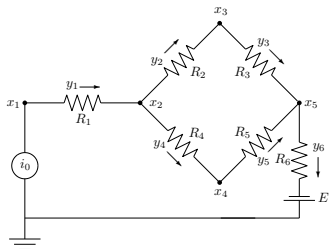
$$y = Ge,$$

Ohm's law

$$A^T y = -f$$

Kirchhoff's current law (KCL)

KVL:



$$e_1 = x_1 - x_2,$$

$$e_2 = x_2 - x_3,$$

$$e_3 = x_3 - x_5,$$

$$e_4 = x_2 - x_4,$$

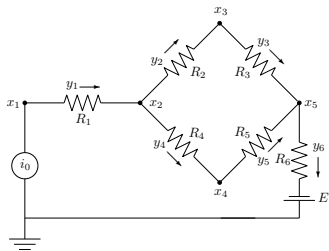
$$e_5 = x_4 - x_5,$$

$$e_6 = x_5 - E.$$

$$\begin{pmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \end{pmatrix} = - \begin{pmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ E \end{pmatrix}$$

## Ohm's Law

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{pmatrix} = \begin{pmatrix} 1/R_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/R_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/R_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/R_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/R_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/R_6 \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \end{pmatrix}$$



KCL:

$$i_0 - y_1 = 0,$$

$$y_1 - y_2 - y_4 = 0,$$

$$y_2 - y_3 = 0,$$

$$y_4 - y_5 = 0,$$

$$y_5 + y_3 - y_6 = 0.$$

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{pmatrix} = - \begin{pmatrix} i_0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Putting it all together:  $A^T G A x = f - A^T G b$ .

$(A^T G)A$

$$\begin{aligned}
 &= \begin{pmatrix} -R_1^{-1} & 0 & 0 & 0 & 0 & 0 \\ R_1^{-1} & -R_2^{-1} & 0 & -R_4^{-1} & 0 & 0 \\ 0 & R_2^{-1} & -R_3^{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & R_4^{-1} & -R_5^{-1} & 0 \\ 0 & 0 & R_3^{-1} & 0 & R_5^{-1} & -R_6^{-1} \end{pmatrix} \begin{pmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix} \\
 &= \begin{pmatrix} R_1^{-1} & -R_1^{-1} & 0 & 0 & 0 & 0 \\ -R_1^{-1} & R_1^{-1} + R_2^{-1} + R_4^{-1} & -R_2^{-1} & -R_2^{-1} & 0 & 0 \\ 0 & -R_2^{-1} & R_2^{-1} + R_3^{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & R_4^{-1} + R_5^{-1} & -R_5^{-1} & 0 \\ 0 & 0 & 0 & -R_5^{-1} & R_3^{-1} + R_5^{-1} + R_6^{-1} & 0 \end{pmatrix}
 \end{aligned}$$

$f - (A^T G)b$

$$\begin{aligned}
 &= \begin{pmatrix} i_0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} -R_1^{-1} & 0 & 0 & 0 & 0 & 0 \\ R_1^{-1} & -R_2^{-1} & 0 & -R_4^{-1} & 0 & 0 \\ 0 & R_2^{-1} & -R_3^{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -R_4^{-1} & 0 \\ 0 & 0 & R_3^{-1} & 0 & R_5^{-1} & -R_6^{-1} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ E \end{pmatrix} = \begin{pmatrix} i_0 \\ 0 \\ 0 \\ 0 \\ E/R_6 \end{pmatrix}
 \end{aligned}$$