CAAM 335 Matrix Analysis: Exam 1

Posted on Friday, 2 October 2008.

Due no later than 5pm on Friday, 9 October 2008.

Instructions:

1. Time limit: **3 uninterrupted hours**.
2. There are six questions worth a total of 100 points.
   Please do not look at the questions until you begin the exam.
3. You may not use any outside resources, such as books, notes, problem sets, friends, calculators, or MATLAB.
4. Please answer the questions thoroughly and justify all your answers. Show all your work to maximize partial credit.
5. Print your name on the line below:

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6. Time started: ____________________ Time completed: ____________________
7. Indicate that this is your own individual effort in compliance with the instructions above and the honor system by writing out in full and signing the traditional pledge on the lines below.

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8. Staple this page to the front of your exam.

Problem 1 (12 points)

Which of the following sets $\mathcal{M} \subset \mathbb{R}^3$ are subspaces? (Justify your answer!)

(a) (4 points) $\mathcal{M} = \{(x_1, x_2, x_3)^T \ : \ x_1 = x_2 = x_3\}$.

(b) (4 points) $\mathcal{M} = \{(x_1, x_2, x_3)^T \ : \ x_1 = x_2,\ x_3 = 1\}$.

(c) (4 points) $\mathcal{M} = \{(x_1, x_2, x_3)^T \ : \ x_1^2 + x_2^2 + x_3^2 \leq 0\}$.

Problem 2 (20 points)

(a) (10 points) Compute the four subspaces $\mathcal{R}(A), \mathcal{N}(A), \mathcal{R}(A^T), \mathcal{N}(A^T)$ associated with

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \end{pmatrix},$$

and give their dimensions.

(b) (5 points) Find all solutions of $Ax = b$, where $b = (1,1)^T$.

(c) (5 points) Find a vector d so that $A^T y = d$ does not have a solution. (Justify your answer!)

Problem 3 (16 points) Always try to justify your answers concisely and rigorously.

(a) (5 points) Compute the inverse of

$$\begin{pmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -3 & -3 & 1 \end{pmatrix}.$$

(b) (5 points) Let I be the identity in $\mathbb{R}^n$. Is it true that the inverse of the matrix

$$I - 2\frac{uu^T}{u^T u}$$

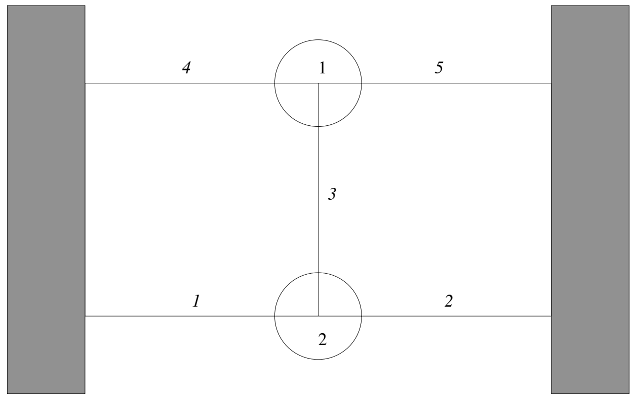
is itself for any nonzero (column) vector $u \in \mathbb{R}^n$?

(c) (6 points) Let $L \in \mathbb{R}^{n\times n}$ be a triangular matrix and let

$$t > \max_{1 \leq i \leq n} |L_{ii}|$$

where L_{ii} is the i-th diagonal element of L. What is the rank of $tI - L$? What are the subspaces $\mathcal{R}(tI - L)$ and $\mathcal{N}(tI - L)$?

Problem 4 (18 points)



(a) (6 points) Compute the adjacency matrix A for the truss shown above.

(b) (6 points) Compute $\mathcal{R}(A)$ and $\mathcal{N}(A)$.

(c) (6 points) Does the linear system $A^T Ax = f$ have solutions for the two right-hand sides below? Justify your answers!

$$f^{(1)} = (1,1,1,-1)^T,$$
$$f^{(2)} = (-1,0,1,0)^T.$$

Problem 5 (16 points) Let A and B be square matrices.

(a) (5 points) Is it necessarily true that $\mathcal{N}(A) \cap \mathcal{N}(B) \subset \mathcal{N}(A+B)$?

(b) (5 points) Show that $\mathcal{N}(B) \subset \mathcal{N}(AB)$.

(c) (6 points) Let A be invertible. Show that $\mathcal{N}(AB) \subset \mathcal{N}(B)$.

Problem 6 (18 points) Let $A \in \mathbb{R}^{m\times n}$ and $B \in \mathbb{R}^{n\times n}$ be nonsingular. Answer the following questions as throughly as possible (proof or counter example).

(a) (6 points) If $\mathcal{R}(A) = \{0\}$ is $A = 0$?

(b) (6 points) Is $\mathcal{R}(A) = \mathcal{R}(AB)$?

(c) (6 points) Is $\mathcal{R}(A) = \mathcal{R}(\beta A)$ for a fixed scalar β?