Problem 0 Follow the instructions on http://www.caam.rice.edu/~yzhang/caam378/software.html and install AMPL and a few solvers. You will need your AMPL installation for this and future homework assignments.

Problem 1 (40 points) Consider the LP

\[
\begin{align*}
\text{min} & \quad 3x_1 + 2x_2, \\
\text{s.t.} & \quad x_1 - x_2 \leq 3, \\
& \quad x_1 - 3x_2 \leq 1, \\
& \quad -4x_1 - x_2 \leq 0, \\
& \quad x_2 \geq 0.
\end{align*}
\]

i. (5 points) Sketch the feasibility set and solve the LP graphically.

ii. (10 points) Use MATLAB’s linprog to solve the LP (turn in the MATLAB script and output generated by MATLAB).

iii. (5 points) Convert the above LP into an LP in the standard form.

iv. (20 points) A basic feasible point of the standard-form feasibility set: \(\{x \in \mathbb{R}^n : Ax = b, x \geq 0\}\), where \(A \in \mathbb{R}^{m \times n} (m < n)\), is such that \(n - m\) of its elements are set to zeros and the remaining \(m\) elements uniquely solve \(Ax = b\) (after eliminating the zero part) and are all nonnegative. Not all zero-nonzero partitions produce basic feasible points since for some partitions the corresponding sub-matrix of \(A\) may be singular or the resulting solution may contain a negative element.

Determine all basic feasible points for the standard-form LP in (iii) and identify their corresponding vertices in the sketched graph for part (i). The determination of all basic feasible points requires checking 20 linear systems (corresponding to different \(m \times m\) sub-matrix of \(A\)) and should be done by a MATLAB loop\(^1\).

Problem 2 (60 points) A small airline, Ivy Air, flies between three cities: Ithaca, Newark, and Boston. They offer several flights but, for this problem, let us focus on the Friday afternoon flight that departs from Ithaca, stops in Newark, and continues to Boston. There are three types of passengers:

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\(^1\)For this small example, you can check the invertibility of an \(m \times m\) submatrix of \(A\), say, \(AB\), using the MATLAB command \(\text{rank}\): 
\[
\text{if}(\text{rank}(AB) < m); \quad \text{fprintf}(''\text{rank}AB < m; \ AB is singular
''); \quad \text{end}.
\]

Instead of using \(x < 0\) to check whether a scalar is smaller than zero, you should use \(x < -\text{eps}\) to avoid problems arising due to round-off errors.
(a) Those traveling from Ithaca to Newark.
(b) Those traveling from Newark to Boston.
(c) Those traveling from Ithaca to Boston.

The aircraft is a small commuter plane that seats 30 passengers. The airline offers three fare classes:

(a) Y class: full coach.
(b) B class: nonrefundable.
(c) M class: nonrefundable, 3-week advanced purchase.

Ticket prices, which are largely determined by external influences (i.e., competitors), have been set and advertised as follows:

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<thead>
<tr>
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<th>IthacaNewark</th>
<th>NewarkBoston</th>
<th>IthacaBoston</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>300</td>
<td>160</td>
<td>360</td>
</tr>
<tr>
<td>B</td>
<td>220</td>
<td>130</td>
<td>280</td>
</tr>
<tr>
<td>M</td>
<td>100</td>
<td>80</td>
<td>140</td>
</tr>
</tbody>
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Based on past experience, demand forecasters at Ivy Air have determined the following upper bounds on the number of potential customers in each of the 9 possible origin-destination/fare-class combinations:

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<th>NewarkBoston</th>
<th>IthacaBoston</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>4</td>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td>B</td>
<td>8</td>
<td>13</td>
<td>10</td>
</tr>
<tr>
<td>M</td>
<td>22</td>
<td>20</td>
<td>18</td>
</tr>
</tbody>
</table>

The goal is to decide how many tickets from each of the 9 origin/destination/fare-class combinations to sell. The constraints are that the plane cannot be overbooked on either of the two legs of the flight and that the number of tickets made available cannot exceed the forecasted maximum demand. The objective is to maximize the revenue.

i. **(30 points)** Formulate this problem as a linear program.

ii. **(30 points)** Use MATLAB’s `linprog` and AMPL to solve the LP (turn in the MATLAB script and output generated by MATLAB, and turn in the AMPL `*.mod`, `*.dat`, `*.run`, and `*.out` (or `*.log`) files).

Generate a table

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which shows how many tickets are sold from each of the 9 origin/destination/fareclass combinations.