section is not applicable. In Chapter 8 we discuss how to recover from this. `nsol` is based on the assumption that a difference Jacobian will be used. In Exercise 5.7.21 you are asked to modify the code to accept an analytic Jacobian. Many times the Jacobian can be computed efficiently by reusing results that are already available from the function evaluation. The parameters m and ρ are assigned the default values of 1000 and .5. With these default parameters the decision to update the Jacobian is made based entirely on the reduction in the norm of the nonlinear residual. `nsol` allows the user to specify a maximum number of iterations; the default is 40.

The Chandrasekhar H-equation. The Chandrasekhar H-equation, [41], [30],

$$F(H)(\mu) = H(\mu) - \left(1 - \frac{c}{2}\int_0^1 \frac{\mu H(\nu)\, d\nu}{\mu + \nu}\right)^{-1} = 0, \tag{5.21}$$

is used to solve exit distribution problems in radiative transfer.

We will discretize the equation with the composite midpoint rule. Here we approximate integrals on $[0,1]$ by

$$\int_0^1 f(\mu)\, d\mu \approx \frac{1}{N}\sum_{j=1}^{N} f(\mu_j),$$

where $\mu_i = (i - 1/2)/N$ for $1 \le i \le N$. The resulting discrete problem is

$$F(x)_i = x_i - \left(1 - \frac{c}{2N}\sum_{j=1}^{N} \frac{\mu_i x_j}{\mu_i + \mu_j}\right)^{-1}. \tag{5.22}$$

It is known [132] that both (5.21) and the discrete analog (5.22) have two solutions for $c \in (0,1)$. Only one of these solutions has physical meaning, however. Newton's method, with the 0 function or the constant function with value 1 as initial iterate, will find the physically meaningful solution [110]. The standard assumptions hold near either solution [110] for $c \in [0,1)$. The discrete problem has its own physical meaning [41] and we will attempt to solve it to high accuracy. Because H has a singularity at $\mu = 0$, the solution of the discrete problem is not even a first-order accurate approximation to that of the continuous problem.

In the computations reported here we set $N = 100$ and $c = .9$. We used the function identically one as initial iterate. We computed all Jacobians with the MATLAB code `diffjac`. We set the termination parameters $\tau_r = \tau_a = 10^{-6}$. We begin with a comparison of Newton's method and the chord method. We present some *iteration statistics* in both tabular and graphical form. In Table 5.1 we tabulate the iteration counter n, $\|F(x_n)\|_\infty / \|F(x_0)\|_\infty$, and the ratio

$$R_n = \|F(x_n)\|_\infty / \|F(x_{n-1})\|_\infty$$