1. **Problem 13.5 from Nocedal and Wright:**

   To find the dual of
   \[
   \min c^T x \quad \text{s.t.} \quad Ax \geq b, \ x \geq 0, \quad (1)
   \]

   we first add a slack variable \( s \) and consider the equivalent LP:
   \[
   \min c^T x \quad \text{s.t.} \quad Ax - s = b, \ x, s \geq 0. \quad (2)
   \]

   Then the dual is the problem of maximizing the minimum of the Lagrangian:
   \[
   \max y \left\{ \min _{x,s \geq 0} c^T x - y^T (Ax - s - b) \right\}. \quad (3)
   \]

   Since
   \[
   c^T x - y^T (Ax - s - b) = (c - A^T y)^T x + y^T s + b^T y,
   \]

   The minimum is \( b^T y \) when \( c - A^T y \geq 0 \) and \( y \geq 0 \), otherwise it is \(-\infty\).

   Therefore, the dual is
   \[
   \max b^T y \quad \text{s.t.} \quad A^T y \leq c, \ y \geq 0. \quad (4)
   \]

2. **Problem 14.8 from Nocedal and Wright:**

   A square matrix is nonsingular if and only if its null space is \( \{0\} \). It suffices to show that the homogeneous system
   \[
   A^T d\lambda + ds = 0, \ Adx = 0, \ Sdx + Xds = 0,
   \]

   only has the trivial solution \( (dx, d\lambda, ds) = (0, 0, 0) \) if and only if \( A \) has full row rank. Clearly, if \( d\lambda = 0 \), then \( ds \) and in turn \( dx \) must be zero. By block Gaussian elimination, we have
   \[
   AXZ^{-1} A^T d\lambda = 0. \quad (5)
   \]

   Since \( x, s > 0 \), the matrix \( AXZ^{-1} A^T \) is symmetric positive semidefinite, and it is definite if and only if \( A \) has full row rank. Therefore, \( d\lambda = 0 \) is the only solution to (5) if and only if \( A \) has full row rank.
3. **Problem 14.9 from Nocedal and Wright:**

(Correction: (14.10) should be (14.16) in the problem.)

The first two block equations are

\[ A^T d\lambda + ds = 0, \quad Adx = 0. \]

Hence, we obtain \( dx^T ds = 0 \) from the following calculation

\[ dx^T (A^T d\lambda + ds) = (Adx)^T d\lambda + dx^T ds = 0^T d\lambda + dx^T ds = 0. \]

4. **Problem 14.11 from Nocedal and Wright:**

To simplify notation, let

\[ (\lambda_+, s_+) = (\lambda, s) + \alpha (d\lambda, ds), \]

where \((d\lambda, ds)\) satisfies

\[ A^T d\lambda + ds = -(A^T \lambda + s). \]

By the linearity,

\[
A^T \lambda_+ + s_+ = (A^T \lambda + s) + \alpha (A^T d\lambda + ds) \\
= (A^T \lambda + s) + \alpha (- (A^T \lambda + s)) \\
= (1 - \alpha)(A^T \lambda + s).
\]

This proves the dual part. The primal part is entirely similar.

5. **Problem 14.17 from Nocedal and Wright:**

Consider (14.39a):

\[
\min_x \frac{1}{2} x^T x, \quad \text{s.t.} \quad Ax = b.
\]

The solution must be unique since it is the shortest solution, in 2-norm, of the underdetermined system \( Ax = b \) where we assume \( A \) to be of full row rank. As the shortest, \( \tilde{x} \) must not contain any component from the null space of \( A \); that is, \( \tilde{x} = A^T u \) is lying entirely in the range of \( A^T \). By substitution, \( A\tilde{x} = AA^T u = b \). Hence \( u = (AA^T)^{-1}b \). As a result,

\[
\tilde{x} = A^T (AA^T)^{-1}b.
\]

On the other hand, since \( s = c - A^T \lambda \), (14.39b) is equivalent to the linear least squares problem

\[
\min_\lambda \frac{1}{2} \| A^T \lambda - c \|^2,
\]

giving \( \tilde{\lambda} = (AA^T)^{-1}Ac \). Hence, \( \tilde{s} = c - A^T \tilde{\lambda} \).