

# Numerical Analysis Part II (CAAM 554)

## Qualifying Exam Preparation List (2011)

**Study Material** (posted on CAAM 554 Website except the textbooks):

- TN-book (textbook: Numerical Linear Algebra by Trefethen and Bau)
- NW-book (textbook: Numerical Optimization by Nocedal and Wright)
- BV-book (Convex Optimization, Chapters 1-2, by Boyd and Vandenbergand)
- CTK-book (Iterative Methods for Optimization, Chapters 1-4, by Kelley)
- Iter-note (Stationary Iterative Methods Notes, by YZ)
- LP-note (A quick intro to LP Note by YZ)
- KS-note (Afternotes on Krylov-Subspace Methods by YZ)
- Diff-note (A Note on Differentiation, an appendix of Tapia's book)

### 1. Iterative Methods for Linear systems

- Stationary iterative methods (Iter-note)
  - Spectral radius  $\rho(A)$ : Definition and relationship to norms
  - When does a stationary iterative method converge?
  - What happens if  $\rho(A) = 1$ ?
  - Diagonal dominance, convergence of Jacobi and Gauss-Seidal
- Arnoldi and Lanzos iterations, GMRES (TB-book, KS-note)
  - Krylov spaces and connection to matrix polynomials
  - What happen upon a breakdown?
  - Worst case scenarios
  - Efficient implementations
- Steepest decent and conjugate gradient methods (TB-book, NW-book)
  - Algorithms: similarity and differences
  - Convergence rates
  - A-conjugacy in CG
  - Other properties of iterates, residuals, search directions, ...
  - Derivations of parameter ( $\alpha$  and  $\beta$ ) formulas

### 2. Unconstrained Optimization and Nonlinear Equations

- Optimality conditions (NW-book, CTK-book)
  - First- and 2nd-order necessary, or sufficient conditions
  - Able to check for simple examples such as a quadratic

- Convexity (BV-book, NW-book)
  - Definition of convex sets and functions
  - Basic properties of convex sets and functions
  - Optimality conditions for minimizing a convex function
  - Able to check for simple examples
- Rate of convergence (NW-book)
  - $q$ -linear, superlinear, quadratic, cubic
  - Definitions, able to check for simple examples
- Line search methods (NW-book, CTK-book)
  - Descent directions
  - Armijo and Wolfe conditions, backtracking
  - Zoutendijk Theorem and ramifications
- Newton's method for optimization and nonlinear systems (NW-book, CTK-book)
  - Standard assumptions
  - Proof of local and Q-quadratic convergence
  - Inexact Newton: basic ideas and convergence rates
- Quasi-Newton methods for optimization (NW-book, CTK-book)
  - Quasi-Newton equation and other basics
  - BFGS and DFP: direct and inverse formulas
  - Positive definiteness and rate of convergence
  - Broyden's method for nonlinear equations
- Nonlinear least squares (NW-book, CTK-book)
  - Gradient and Hessian formulas
  - Gauss-Newton and Levenberg-Marquardt methods
  - Convergence rate, Gauss-Newton under zero-residual
- Linear programming (NW-book, LP-note)
  - Basics of polyhedra, vertices of convex sets
  - Standard form LP and its dual
  - Weak and strong duality (able to prove the former)
  - Optimality conditions (how they differs from unconstrained)
  - Primal-dual interior-point method basics