CAAM 565: HW 10 (F2019)

Given real numbers x and $\lambda > 0$, consider solving the non-smooth convex optimization problem

$$t^* = \arg\min_t f(t) = \lambda|t| + \frac{1}{2}(t - x)^2.$$

We know that t^* satisfies

$$0 \in \partial f(t^*),$$

where $\partial f(t)$ is the sub-differential of f at t. It is clear that t^* is a function of x and λ. We will denote it as

$$t^* = S_\lambda(x).$$

Assignment: (Due on November 26 in class)
(1) Derive a formula for $S_\lambda(x)$. You derivation should be as rigorous as possible and the formula should be as simple as possible. Try condensing your formula into a one-liner if you can (hint: use the *sign* function and more).
(2) Submit a one-page, typed sheet for your formula and derivation.