CAAM 565: HW 3, PROBLEM 2

F2019

Let $\mathcal{K} \subset \mathbb{R}^n$ be a proper cone and $\mathcal{K}^*$ be its dual cone. Clearly, $\mathcal{K}^*$ is convex and closed. It is also solid and pointed; hence $\mathcal{K}^*$ is a proper cone.

Let the pair of points $x, y \in \mathbb{R}^n$ and $u, v \in \mathbb{R}^n$ satisfy, respectively,

$$x \succeq_{\mathcal{K}} y \succeq_{\mathcal{K}} 0, \quad \text{and} \quad u \succeq_{\mathcal{K}^*} v \succeq_{\mathcal{K}^*} 0.$$

Prove that

$$x^T u \geq y^T v. \tag{1}$$