CAAM 565: L1 Regularization

Two Optimization Models

We consider recovering a desired signal \( \hat{x} \in \mathbb{R}^n \) that approximately satisfies an under-determined linear system \( Ax = b \in \mathbb{R}^m \), where \( m < n \), with the help of a regularization function \( \phi(Dx) \) where

\[
\phi(x) = \|x\|_1
\]  

(1)

is sparsity promoting, and \( D \) is a finite difference matrix of either the zeroth (i.e., identity), or the first, or the second order. Clearly, \( \phi(x) \) is a non-smooth function, not differentiable whenever \( x \) has a zero element.

The first model is

\[
\min_{x \in \mathbb{R}^n} \phi(Dx), \text{ s.t. } Ax = b,
\]  

(2)

which can be converted into a linear program and solved by Matlab function \texttt{linprog}.

The second model is an unconstrained minimization model

\[
\min_{x \in \mathbb{R}^n} \phi_\sigma(Dx) + \frac{\mu}{2}\|Ax - b\|_2^2,
\]  

(3)

where

\[
\phi_\sigma(x) = \sum_{i=1}^n \sqrt{x_i^2 + \sigma}
\]  

(4)

is a smooth, differentiable approximation to \( \phi(x) \) for a small parameter \( \sigma > 0 \).

Assignment

Write two Matlab functions

\[
\text{x} = \text{myL1reg}(A, b, D);
\]

\[
\text{x} = \text{myL1regl}(A, b, D);
\]

The first function uses \texttt{linprog} to solve the model (2). In the second function, you implement a gradient method using back-tracking line search to solve the model (3), where the parameters \( \mu \) and \( \sigma \) should be chosen carefully by you.

Further details can be found at the course website.