

CAAM 335: Matrix Analysis

HW 1, August 26, 2009

Due: September 2, 2009

Problem 1 (20 points) Let

$$A = \begin{pmatrix} 2 & 3 & 5 \\ 1 & 0 & 3 \\ 6 & 2 & 3 \end{pmatrix}.$$

- 1.1 Compute Ax where $x = (2, 1, -3)^T$.
- 1.2 Write Ax as a linear combination of columns of A .
- 1.3 Compute the square root of the inner product of Ax with itself. This quantity is called the 2-norm of the vector (Ax , in this case) which measures the magnitude of the vector.

Problem 2 (30 points) Let us consider the following matrices

$$A = \begin{pmatrix} 1 & 2 \\ 3 & -4 \end{pmatrix}, \quad B = \begin{pmatrix} 5 & 0 \\ -6 & 7 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & -3 & 4 \\ 2 & 6 & -5 \end{pmatrix}.$$

- 2.1 Find $5A - 2B$.
- 2.2 Find: AB and BA . Note that $AB \neq BA$.
- 2.3 Find: (a) AB and $(AB)C$; (b) BC and $A(BC)$. Note that $(AB)C = A(BC)$.
- 2.4 Find A^T , B^T , and $A^T B^T$. Note that $A^T B^T \neq (AB)^T$.
- 2.5 Find $AA = A^2$, and AC .

Problem 3 (10 points) Consider A and B in Problem 2. Write AB into a sum of vector outer products. (Hint: partition A by columns and B by rows.)

Problem 4 (20 points) (Problem 1 on page 9 of the Lecture Notes)

In order to refresh your matrix-vector multiply skills please calculate, by hand, the product $A^T GA$ in the 3 compartment case and write out the 4 equations in (1.1). The second equation should read

$$\frac{-x_1 + 2x_2 - x_3}{R_i} + \frac{x_2}{R_m} = 0.$$