CAAM 335 Matrix Analysis: HW 10

Problem 1 (20 points) Consider

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

(a) (6 points)
Compute all the eigenvalues and orthonormal eigenvectors of AA^T and A^TA (many quantities can be easily obtained without substantial calculations).

(c) (7 points)
Find the singular value decomposition $A = U\Sigma V^T$ by hand and show your steps (you may compare with the result from the MATLAB command `svd`).

(d) (7 points)
Without any further computation, give an orthonormal basis for each of the subspaces: $\mathcal{R}(A^T)$, $\mathcal{N}(A)$, $\mathcal{R}(A)$, $\mathcal{N}(A^T)$, if it is non-trivial.

Problem 2 (10 points) The pseudo-inverse of A is defined as $A^\dagger = V\Sigma^\dagger U^T$. Is it true or false that $AA^\dagger A = A$ and $A^\dagger AA^\dagger = A^\dagger$? Justify your answers.

Problem 3 (10 points) Let matrix $A \in \mathbb{R}^{m\times n}$. Prove that both $AA^\dagger$ and $A^\dagger A$ are orthogonal projection matrices (idempotent and symmetric). Which subspaces they project onto, respectively? Justify your answers.

Problem 4 (10 points) Let matrix $A \in \mathbb{R}^{m\times n}$ have rank n. Show that in this case $A^\dagger = (A^TA)^{-1}A^T$ and $A^\dagger A = I$ (so $A^\dagger$ is also called left inverse). Give the analogous results for the case when A has rank m.