Homework 6 Posted on Tuesday 04/03/2012 and due on Thursday 04/12/2012.

• Problems for 402 and 502

1. Let $A$ be a closed rectangle in $\mathbb{R}^n$ and $C$ a proper subset of $A$. Prove that for all partitions $P$ of $A$,

$$V(C, P) - v(C, P) = V(\partial C, P).$$

2. With $A$ and $C$ as in problem 1, let $P$ be a partition of $A$ and $P'$ a refinement of $P$. Prove that

$$0 \leq v(C, P) \leq v(C, P') \leq V(C, P') \leq V(C, P).$$

3. Suppose that $A$ is a rectangle and that functions $f : A \to \mathbb{R}$ and $g : A \to \mathbb{R}$ are integrable on $A$. Prove that $fg$, the product function defined pointwise on $A$ by $fg(x) = f(x)g(x)$ is integrable.

4. Prove that finite unions and intersections of Jordan measurable sets are Jordan measurable. Prove also that if $A$ and $B$ are two Jordan measurable sets, $A \setminus B$ is also Jordan measurable.

• Problems for 502

1. Prove that an unbounded set cannot have content zero. Give an example of a closed set of measure zero that does not have content zero.

2. If $C$ is a set of content zero, prove that the boundary of $C$ has content zero. Give an example of a bounded set of measure zero such that its boundary does not have measure zero.