CAAM 651 · Topics in Numerical Linear Algebra
SPECTRA AND PSEUDOSPECTRA
Spring 2003 · Rice University

Lectures: Tuesday/Thursday 1PM, Keck Hall 101

Web Sites: http://www.caam.rice.edu/~embree/651
http://www.comlab.ox.ac.uk/pseudospectra (The Pseudospectra Gateway)

Instructor: Mark Embree (embree@rice.edu)
Duncan Hall 2006, (713) 348-6160

Office Hours: Monday 1–2PM, Thursday 3–5PM, or by appointment.

Prerequisite: A good background in matrix theory (e.g., CAAM 335).
Skills in numerical linear algebra (CAAM 451/453/551) will be helpful, but are not essential.

Grading: Grades will be based on an active participation in this seminar,
and a project consisting of a twenty-minute presentation and a short (e.g., 4 page) report.

Course Notes: Spectra and Pseudospectra: The Behavior of Nonnormal Matrices and Operators
by Lloyd N. Trefethen and Mark Embree. Draft chapters will be distributed
throughout the semester. Your feedback and corrections are most welcome.

Optional Text: Perturbation Theory for Linear Operators by Tosio Kato
We will not follow this wonderful book closely, though it is a great complement
to the topics we will discuss. The first chapter, a review of matrix theory,
is especially elegant, if concise. Students considering research in eigenvalue
problems are encouraged to buy this book.

Any student with a disability requiring accommodation in this course is encouraged
to contact the instructor during the first week of class, and also to contact
Disability Support Services in the Ley Student Center.
Eigenvalues arise in many applications, ranging from structural engineering, quantum mechanics, and fluid dynamics to population biology, ecology, probability, and economics. A number of these applications give rise to symmetric matrices, whose eigenvalues are relatively straightforward to analyze and compute. In particular, small changes to the matrix (as can occur, for example, due to model uncertainties, finite precision computations, etc.) do not change the eigenvalues much; we say that the eigenvalues of a symmetric matrix are well-conditioned.

In a number of other applications, the matrices of interest are not symmetric. In some cases, the eigenvalues of such matrices are very sensitive to changes in the matrix, and may be very difficult to compute accurately; we say that such eigenvalues are ill-conditioned. But this sensitivity to perturbations is more than just a computational inconvenience; eigenvalues are interesting because they have some physical meaning, but physical conclusions drawn from highly sensitive eigenvalues can be misleading, or even wrong altogether!

In this seminar, we will discuss a theory that has blossomed over the past fifteen years or so, known as pseudospectra. This tool provides techniques for analyzing eigenvalue sensitivity and predicting the implications of such sensitivity for a given application. We will study the mathematical background of the subject, applications in numerical analysis, and applications in various fields of physics, probability, and biology.

CAAM 651 · Rough Course Outline

1. Review of Matrix Theory and the Source of Eigenvalues in Applications
2. Introduction to Pseudospectra
3. Mathematical Properties of Pseudospectra
4. Computation of Pseudospectra
5. Analysis of Iterative Methods for Linear Systems
6. Analysis of Numerical Methods for Differential Equations
7. Spectral Theory of Toeplitz Matrices
8. Pseudospectra of Differential Operators
9. Applications in Fluid Dynamics
10. Applications in Control Theory
11. Applications in Probability Theory
12. Applications in Ecology and Population Biology