Office Hours

• My office hours
  – Rm 435, Humanities, Tuesdays from 1:30pm to 3:00pm
  – Rm 435, Humanities, Thursdays from 1:30pm to 3:00pm

• Tom Hunt is the TA for this class. His lab hours are now as follow

  – SCSI 1004, Tuesdays from 3:30 until 4:45
  – SCSI 1004, Wednesdays from 12:00 until 12:50
  – Hum 346 on Wednesdays from 2:30-3:30
Application: One-Dimensional Electrostatic Motion
Charge Repulsion

• Now we will consider the case of charged particles with the same sign charge

• Instead of attracting each other, the charges repel each other.
Particle Accelerations

Each particle has a vector acceleration directly away from the other particle.
Team Project

Q1) Modify the planets2.m and forcing.m to simulate the following:

- There are $N$ electrically charged particles confined to move in the x-direction only
- Distribute the charges initially at equispaced points in $[-1,1]$
- The equations of motion of the charges are:

\[
\begin{align*}
      x_1(t) &= -1, \quad v_1(t) = 0 \\
      x_N(t) &= 1, \quad v_N(t) = 0 \\

      \frac{dx_n}{dt} &= v_n \\
      \frac{dv_n}{dt} &= \sum_{i=1, i \neq n}^{i=N} \frac{-(x_i - x_n)}{(x_i - x_n)^3}
\end{align*}
\]
Big Team Project

• Ok – we have been warming up with some small projects

• Now for something a little more involved
The Physics

• The domain of interest is a two-dimensional periodic box of side length L

• It is populated with N circles of mass M each and radius R
Domain

The domain of the function is defined by the rectangular region with boundaries:
- $x = 0$
- $x = L$
- $y = 0$
- $y = L$
Defining A Set of Motion Rules

• We are now going to define a set of rules governing the motion of the disks
Rule 1

- The disks are initially randomly distributed

- The disks have random initial velocity (with the absolute value of each component of velocity bounded by 1)
Rule 2

• Effectively Newton’s first:
  – all particles do not accelerate until they collide
Particles Collide

Before

\[
\begin{pmatrix}
  u_2^- \\
  v_2^-
\end{pmatrix}
\]

\[
\begin{pmatrix}
  u_1^- \\
  v_1^-
\end{pmatrix}
\]

After

\[
\begin{pmatrix}
  u_2^+ \\
  v_2^+
\end{pmatrix}
\]

\[
\begin{pmatrix}
  u_1^+ \\
  v_1^+
\end{pmatrix}
\]
Rule 4

• Momentum is conserved in a collision
Rule 5

- Angular momentum is conserved in the collision
Rule 6

• Total kinetic energy is conserved

• The total kinetic energy of the particles is the same before and after the collision.
Model Simplification

• We are going to use Rules 4, 5, and 6 to determine the change in velocity of two disks when they collide.

• We can simplify this problem by considering a coordinate frame which is:
  – co-moving with the center of one of the disks (say disk1) at its origin
  – with x-axis aligned in direction of the segment connecting the two disk centers
In Co-Moving Frame

Before

\[
\begin{pmatrix}
  u_2^- \\
  v_2^- \\
\end{pmatrix} = \begin{pmatrix}
  u_1^- = 0 \\
  v_1^- = 0 \\
\end{pmatrix}
\]
Math Version of 4
(in co-moving frame)

• Conservation of \textit{linear momentum}:

\[
M \begin{pmatrix} 0 \\ 0 \end{pmatrix} + M \begin{pmatrix} u_-^2 \\ v_-^2 \end{pmatrix} = M \begin{pmatrix} u_1^+ \\ v_1^+ \end{pmatrix} + M \begin{pmatrix} u_2^+ \\ v_2^+ \end{pmatrix}
\]

• Which reduces to:

\[
\begin{align*}
u_-^2 &= u_1^+ + u_2^+ \\
v_-^2 &= v_1^+ + v_2^+
\end{align*}
\]
Math Version of 5
(in co-moving frame)

• Conservation of \textit{angular} momentum:

\[ Mx_2^- v_2^- = Mx_2^+ v_2^+ \]

• And since the center of two can not be at the origin this implies:

\[ v_2^- = v_2^+ \]
Math Version of 6
(in co-moving frame)

• Conservation of kinetic energy:

\[ \frac{M}{2} \left( (u_2^-)^2 + (v_2^-)^2 \right) = \frac{M}{2} \left( (u_1^+)^2 + (v_1^+)^2 + (u_2^+)^2 + (v_2^+)^2 \right) \]

• Reduces to:

\[ (u_2^-)^2 + (v_2^-)^2 = (u_1^+)^2 + (v_1^+)^2 + (u_2^+)^2 + (v_2^+)^2 \]
Summary of 4,5,6

\[ u_2^- = u_1^+ + u_2^+ \]
\[ v_2^- = v_1^+ + v_2^+ \]
\[ v_2^- = v_2^+ \]
\[ (u_2^-)^2 + (v_2^-)^2 = (u_1^+)^2 + (v_1^+)^2 + (u_2^+)^2 + (v_2^+)^2 \]
Solutions

**Collision**

\[
\text{if } (u_2^- \geq 0) \\
\begin{align*}
u_1^+ &= u_2^- \\
\nu_1^+ &= 0 \\
n_2^- &= 0 \\
n_2^- &= 0
\end{align*}
\]

**No collision**

\[
\text{if } (u_2^- < 0) \\
\begin{align*}
u_1^+ &= 0 \\
\nu_1^+ &= 0 \\
n_2^+ &= u_2^- \\
n_2^+ &= 0
\end{align*}
\]
In Co-Moving Frame

Before

\[
\begin{pmatrix}
  u_2^- \\
  v_2^-
\end{pmatrix}
= \begin{pmatrix}
  0 \\
  v_2^-
\end{pmatrix}
\]

Before

\[
\begin{pmatrix}
  u_1^- = 0 \\
  v_1^- = 0
\end{pmatrix}
\]

\[
\begin{pmatrix}
  u_1^+ = u_2^- \\
  v_1^- = 0
\end{pmatrix}
\]
Reverting To Non-moving Frame

The rules essentially say that if two equal mass disks collide then:

1) they exchange velocity in their center-to-center vector

2) they retain their own velocity in the direction of the tangent to their center-to-center vector
General Formula

\[
\begin{aligned}
(u_1^+ & ) = (u_1^-) + ((u_2^- - u_1^-) \cdot (x_2^- - x_1^-)) \\
& \quad \cdot \frac{(x_2^- - x_1^-)}{(x_2^- - x_1^-)^2 + (y_2^- - y_1^-)^2}
\end{aligned}
\]

\[
\begin{aligned}
(v_1^+ & ) = (v_1^-) + ((u_2^- - u_1^-) \cdot (y_2^- - y_1^-)) \\
& \quad \cdot \frac{(-x_2^- + x_1^-)}{(x_2^- - x_1^-)^2 + (y_2^- - y_1^-)^2}
\end{aligned}
\]
Summary of Collision Formula

• Given the relative position of two disks before the collision and their velocities we can determine the post-collision velocities
\[
\begin{align*}
\begin{pmatrix}
  u_1^+ - u_1^- \\
  v_1^+ - v_1^-
\end{pmatrix}
&=
\begin{pmatrix}
  u_2^- - u_1^- \\
  v_2^- - v_1^-
\end{pmatrix}
\cdot
\begin{pmatrix}
  x_2^- - x_1^- \\
  y_2^- - y_1^-
\end{pmatrix}
\frac{
\begin{pmatrix}
  x_2^- - x_1^- \\
  y_2^- - y_1^-
\end{pmatrix}
}{
(x_2^- - x_1^-)^2 + (y_2^- - y_1^-)^2
}\end{align*}
\]

Change in velocity of disk 1

Exchange of linear and angular momentum
Project Description

• Using Euler forward as time stepping
• Initiate the random initial positions and velocities of the disks
• Move the particles with time step dt
• Each time step check to see if any disk intersects any other disk
  – if two disks intersect use the collision formula to determine their new velocities
• Increment the disk positions with dt multiplying their velocities
Project Description cont

• Use \textit{mod} on the updated positions to make sure that the disks stay within the box

• Repeat until quite a few collisions show up

• Plot the locations of the particles in a two-dimensional plot – use \textit{hold} so that the time history of the disks shows up
Project Description cont

- Work in groups

- Hand in on 09/30/02

- Each individual must hand in their own report (following the syllabus guidelines)

- Include – write up, code, figures of disk paths

- Try not to use loops too much (looping is slow)