CAAM/NEUR 415: EXAMINATION # 1
February 27, 2004

Instructions:

(1) Time limit: 3 contiguous hours.

(2) No outside sources. That means no books, no notes, no computer and no calculator.

(3) Work each problem in great detail. There are 4 problems.

(4) `Print your name on the line below`

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(5) Indicate your compliance with the honor system by writing out in full and signing the traditional pledge on the lines below

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(6) **Staple** this cover sheet to your solutions and return it to Steve Cox by 5 pm on Tuesday, March 9.

1. Together, we shall study, by hand, the passive sealed fiber undergoing a known distributed current stimulus, I. That is,

$$\begin{aligned}
&\frac{a}{2R_2}v_{xx} = C_m v_t + G_m v + I(x,t), \quad 0 < x < \ell \\
&v_x(0,t) = v_x(\ell,t) = 0 \\
&v(x,0) = 0.
\end{aligned} \tag{1}$$

We will solve this at first for a general I and then finish up with a particular case.

(a) **[6 points]**We shall expand everything in terms of the solutions to

$$-q''(x) = zq(x), \quad z'(0) = z'(\ell) = 0. \tag{2}$$

Solve for the eigenpairs $\{q_n, z_n\}_{n=0}^{\infty}$ of (2) and normalize the eigenfunctions in order that $\int_0^\ell q_n^2(x)\,dx = 1$.

(b) **[12 points]** Now write

$$v(x,t) = \sum_{n=0}^{\infty} q_n(x)T_n(t) \quad \text{and} \quad I(x,t) = \sum_{n=0}^{\infty} q_n(x)i_n(t)$$

and plug these into (1) and derive a sequence of ordinary differential equations (and initial conditions) for the unknown T_n in terms of the known i_n.

(c) **[8 points]** Solve these ordinary differential equations.

(d) **[6 points]** Explicitly compute each i_n in the case that $I(x,t) = t\exp(-t)\cos(\pi x/\ell)$. What are the corresponding T_n? And finally, what is v?

2. If the stimulus above was a conductance change, rather than a direct current stimulus we would instead be faced with

$$\begin{aligned}
&\frac{a}{2R_2}v_{xx} = C_m v_t + G_m v + G(x,t)(v-E), \quad 0 < x < \ell \\
&v_x(0,t) = v_x(\ell,t) = 0 \\
&v(x,0) = 0.
\end{aligned} \tag{3}$$

you might think of G as some effective synaptic conductance and E as the (constant) synaptic reversal potential. As our eigentechnique of problem 1 does not directly apply we turn to a strictly numerical attack.

(a) **[14 points]** Equipartition the fiber into N equipotential compartments and show how (3) becomes a system of N ordinary differential equations for the N compartment potentials. Write this in matrix terms.

(b) **[10 points]** Setup both forward and backward Euler time discretizations of this system of ordinary differential equations.

3. We now return to the eigentechnique but in a context that we neglected in lecture. In particular, suppose we have a mother fiber that forks into two daughters. We will call the mother branch 1 and daughters 2 and 3. For simplicity we will suppose that they are each of unit length and radius and that they are sealed at their tips. In this case, the eigenproblem takes the form

$$\begin{aligned}
&-q''_{n,1}(x) = z_n q_{n,1}(x), \quad -q''_{n,2}(x) = z_n q_{n,2}(x) \quad \text{and} \quad -q''_{n,3}(x) = z_n q_{n,3}(x) \\
&q_{n,1}(1) = q_{n,2}(0) = q_{n,3}(0) \\
&q'_{n,1}(1) = q'_{n,2}(0) + q'_{n,3}(0) \\
&q'_{n,1}(0) = q'_{n,2}(1) = q'_{n,3}(1) = 0
\end{aligned}$$

You see there is a common eigenvalue, z_n, for each of the branches but that the associated eigenfunction has 3 "pieces," namely $q_{n,1}$, $q_{n,2}$ and $q_{n,3}$. These are each functions on the unit interval, $[0,1]$, subject to end and matching conditions.

(a) **[14 points]** Draw a picture of this forked cell and offer detailed physical interpretations of each of the end/matching conditions above. In particular, what does $q_{n,1}(1) = q_{n,2}(0) = q_{n,3}(0)$ mean? What does $q'_{n,1}(1) = q'_{n,2}(0) + q'_{n,3}(0)$ mean? And what do the three equations $q'_{n,1}(0) = q'_{n,2}(1) = q'_{n,3}(1) = 0$ mean?

(b) **[10 points]** Notice that $z_0 = 0$, $q_{0,1} = q_{0,2} = q_{0,3} = 1$ satisfies this system. Please exhibit and interpret at least 3 other eigensolutions (with nonconstant eigenfunctions).

4. **[20 points]** Our stated goal was a quantitative understanding of the cartoon of synaptic plasticity. The cartoon's legend contains the keywords: calcium, depolarization, calcium channel, AMPA receptor, NMDA receptor, synaptic vesicle and magnesium. Please provide a **detailed** narration of the cartoon, using each of these keywords, that describes how a presynaptic signal may lead to strengthening of the postsynaptic conductance. Describe, at the proper moment, how action potentials are generated, the directions in which they travel, and the means by which (and the reasons for) the cell may possess a strongly excitable cell body yet a weakly excitable dendritic tree.