CAAM 440 · APPLIED MATRIX ANALYSIS

Problem Set 4

Posted Friday 9 March 2012. Due Monday 19 March 2012.

This assignment asks you to read and report upon a recent survey article on *tensors*, a generalization of vectors and matrices that has become increasingly important prominent in the matrix theory community over the past decade.

Download the article: "Tensor Decompositions and Applications" by Tamara G. Kolda and Brett W. Bader, *SIAM Review* 51 (2009) 455–500: http://dx.doi.org/10.1137/07070111X.

Read the article, paying particular attention to Sections 1, 2, 3, 4, 5.7, 6, 7 (i.e., skim most of Section 5).

Write-up a digest of this article. Your report should conform to the following specifications.

1. Type your report (using LaTeX or your favorite word processor).
2. The report should be at least two pages and *no more than five pages*.
3. You may work alone, or with one other student. (In the latter case, submit one report, and clearly indicate the names of both authors.)
4. *Do not quote at length from the article.* Read, digest, and explain in your own words.
5. Address the following points:
   (a) What is a tensor?
   (b) Define key terms, such as *fiber*, *slice*, *mode*, *rank-one tensor*, etc.
   (c) Describe the CANDECOMP/PARAFAC decomposition, explaining how it is a generalization of the "dyadic form" of the singular value decomposition,
   $$\mathbf{A} = \sum_{j=1}^{r} \sigma_j \mathbf{u}_j \mathbf{v}_j^*.$$
   What is the *rank* of a tensor, and why is it difficult to calculate? Is the decomposition unique? How (very roughly) is it calculated?
   (d) How does the Tucker decomposition differ from the CP decomposition? What is the HOSVD? Is it easier or harder to calculate than the CP decomposition?
   (e) For matrices, truncating the SVD after k terms gives the optimal rank-k approximation to a matrix. Is the same true for the CP or HOSVD tensor factorizations?
   (f) What part of the article did you find most interesting?
   (g) What part of the article did you find difficult to understand?
   (h) Go beyond the paper in some way: for example, track down one of the many citations to the applications literature, and briefly describe how tensors are used in an area that interests you; or download the MATLAB "Tensor Toolbox" and do some numerical calculations. You could numerically verify the claim on page 465 about the percentage of $2 \times 2 \times 2$ tensors that have rank 2 or 3, of you could construct a numerical example to illustrate the best-approximation example on page 469.

Grading: This assignment will be weighted equally with the other unpledged problem sets.
You will be graded as follows: (a) = 5 points, (b) = 10 points, (c) = 15 points, (d) = 15 points, (e) = 10 points, (f) = 5 points, (g) = 5 points, (h) = 15 points; quality of writing and clarity of exposition: 20 points.