Reduced Basis Methods for Dynamical Systems

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LTI Systems and Model Reduction

Time Domain

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \\ \mathbf{y} &= \mathbf{C}\mathbf{x} \end{aligned}$$

$$\mathbf{A} \quad \in \mathbb{R}^{n \times n}, \quad \mathbf{B} \in \mathbb{R}^{n \times m}, \quad \mathbf{C} \in \mathbb{R}^{p \times n}, \quad n >> m, p$$

Frequency Domain

$$\begin{aligned} s\mathbf{x} &= \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \\ \mathbf{y} &= \mathbf{C}\mathbf{x} \end{aligned}$$

Transfer Function

$$\mathbf{H}(s) \equiv \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}, \qquad \mathbf{y}(s) = \mathbf{H}(s)\mathbf{u}(s)$$

Model Reduction

Construct a new system $\{\hat{\mathbf{A}}, \hat{\mathbf{B}}, \hat{\mathbf{C}}\}$ with LOW dimension $k << n$

$$\begin{aligned} \dot{\hat{\mathbf{x}}} &= \hat{\mathbf{A}}\hat{\mathbf{x}} + \hat{\mathbf{B}}\mathbf{u} \\ \hat{\mathbf{y}} &= \hat{\mathbf{C}}\hat{\mathbf{x}} \end{aligned}$$

Goal: Preserve system response

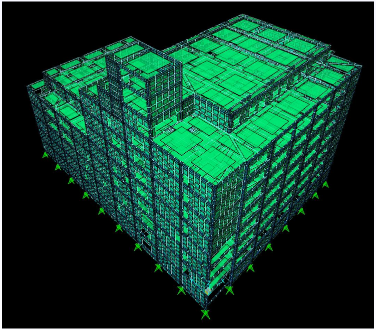
$$\underline{\hat{\mathbf{y}} \ \text{ should } \ \text{approximate } \ \mathbf{y}}$$

Projection: $\mathbf{x}(t) = \mathbf{V}\hat{\mathbf{x}}(t)$ and $\mathbf{V}\dot{\hat{\mathbf{x}}} = \mathbf{A}\mathbf{V}\hat{x} + \mathbf{B}\mathbf{u}$

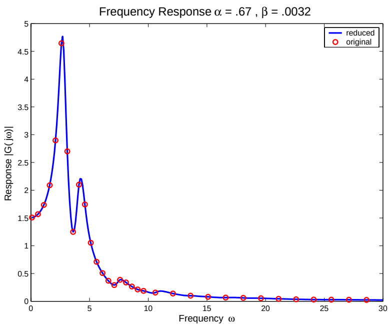
Applications

Mechanical Systems | Electrical Systems | MEMS devices

e.g. Building Model



$N = 26394,\ k = 200$ (ROM)



The Symmetric SVD Approximation

If $\mathbf{W}\mathbf{X}_2 = \mathbf{X}_1 + \mathbf{E}$ where $\mathbf{W} = \mathrm{blockdiag}(\mathbf{I} - 2\mathbf{w}\mathbf{w}^T)$

$$\min_{\mathbf{W}\hat{\mathbf{X}}_2 = \hat{\mathbf{X}}_1} \left\| \begin{pmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \end{pmatrix} - \begin{pmatrix} \hat{\mathbf{X}}_1 \\ \hat{\mathbf{X}}_2 \end{pmatrix} \right\|_F^2 \quad \text{and} \quad \begin{pmatrix} \hat{\mathbf{X}}_1 \\ \hat{\mathbf{X}}_2 \end{pmatrix} = \mathbf{U}\mathbf{S}\mathbf{V}^T$$

Solved by:

$$\mathbf{U} = \frac{1}{\sqrt{2}} \begin{pmatrix} \mathbf{U}_1 \\ \mathbf{U}_2 \end{pmatrix}, \quad \mathbf{S} = \sqrt{2}\mathbf{S}_1, \quad \mathbf{V} = \mathbf{V}_1. \quad \text{and} \quad \mathbf{U}_2 = \mathbf{W}\mathbf{U}_1,$$

with

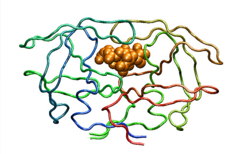
$$\boxed{\mathbf{U}_1\mathbf{S}_1\mathbf{V}_1^T = \tfrac{1}{2}(\mathbf{X}_1 + \mathbf{W}\mathbf{X}_2)}$$

Animation: Rotationally Symmetric SVD

click figure for movie

Animation: Rotationally Symmetric SVD on HIV1

click for movie



Front View

First SVD mode – Rotationally Symmetric vs. Unsymmetric

Red = Unsymmetric   Blue = Symmetric